

The thermodynamics of the quark-gluon plasma: Self-consistent resummations vs. lattice data

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We discuss a recent approach for overcoming the poor convergence of the perturbative expansion for the thermodynamic potential of QCD. This approach is based on self-consistent approximations which allow for a gauge-invariant and manifestly ultraviolet-finite resummation of the essential physics of the hard thermal/dense loops. The results thus obtained are in good agreement with available lattice data down to temperatures of about twice the critical temperature. Calculations for a plasma with finite quark density (i.e., with a non-zero chemical potential μ) are no more difficult than at $\mu = 0$.

1. The failure of the conventional perturbation theory

The most compelling theoretical evidence for the existence of the quark-gluon plasma comes from lattice QCD which shows a clear signal for a deconfinement phase transition [1]. Above it, the lattice results slowly approach the ideal-gas limit from below, with important deviations, though, of about 15-20%, up to temperatures $\sim 5T_c$, and these are expected to remain noticeable ($\sim 10\%$) even at temperatures as high as $10^3 T_c$ [2,3].

This suggests a picture of the high-temperature phase of QCD where the interactions are more important than one would naïvely expect on the basis of the asymptotic freedom alone, but where the effects of these interactions are nevertheless small enough to be computable via weak-coupling techniques.

The weak coupling expansion of the thermodynamic potential \mathcal{F} (free energy) is presently known [4] to order $\alpha_s^{5/2}$, or g^5 ($\alpha_s \equiv g^2/4\pi$), but it shows a disappointingly poor convergence except for coupling constants $\alpha_s \lesssim 0.05$ (which would correspond to temperatures $\gtrsim 10^5 T_c$). Already the next-to-leading order correction of $\mathcal{O}(g^3)$ signals the inadequacy of the conventional perturbation theory except for very small coupling, because, in contrast to the leading-order terms, it leads to a free energy in excess of the ideal-gas value.

This blatant inadequacy is somewhat surprising, since one expects the free energy to be dominated by the *hard* thermal fluctuations with momenta $k \sim T$, for which perturbation theory should apply. Indeed, at temperatures $T \gtrsim T_c \sim 300$ MeV, the QCD coupling is reasonably small, $\alpha_s \lesssim 0.3$, when renormalized at the Matsubara scale $\bar{\mu} = 2\pi T$.

But a closer inspection of the perturbative expansion reveals that this is truly an expansion in powers of g (rather than α_s), with $g \sim 1$ for all temperatures of interest, and that the largest “corrections” are associated with *odd* powers of g . The latter come from

resummations which take into account the phenomenon of Debye screening at the scale gT [5]. Thus, the large perturbative corrections are actually associated with *soft* degrees of freedom, with momenta of order gT , and arise when the contribution of the soft modes to the free energy is expanded in powers of g . But this expansion is potentially troublesome, since, as we shall shortly recall, the soft modes are non-perturbative.

2. The quasiparticle picture of the quark-gluon plasma

The soft degrees of freedom are collective excitations which would not even exist in the absence of interactions [5]. To leading order in g , their dynamics is described by an effective theory obtained by integrating out the “hard” ($k \sim T$) thermal fluctuations to one-loop order. This generates a set of non-local self-energy and vertex amplitudes, known as “hard thermal loops” (HTL) [6], which encompass screening effects and non-trivial dispersion relations. At momenta $k \lesssim gT$, the HTL’s are *leading-order* effects and must be resummed for consistency [6]. Thus, when computing the contribution of the soft modes to thermodynamical functions, these modes should be treated as dressed *quasiparticles*, with properties described by the HTL effective theory. This suggests a description of the thermodynamics of hot QCD in terms of weakly interacting “hard” and “soft” quasiparticles (rather than the more strongly interacting elementary quanta). This is also supported by the success of phenomenological fits involving simple massive “quasiparticles” in reproducing the lattice results [7].

Quite generally, the physical information on the quasiparticles is contained in the spectral density $\rho(\omega, k)$ related to the corresponding propagator by:

$$D(\omega, k) = \int_{-\infty}^{\infty} \frac{dk_0}{2\pi} \frac{\rho(k_0, k)}{k_0 - \omega}. \quad (1)$$

For free massless excitations, $\rho_0(\omega, k) \propto \delta(\omega^2 - k^2)$. In the HTL approximation, the spectral densities are divided into a pole piece at time-like momenta, and a continuum piece at space-like momenta. At soft momenta $k \lesssim gT$, all the pieces of the HTL spectral functions appear to be equally important in our numerical calculations of thermodynamical quantities [2]. At large momenta, the HTL spectral densities take the approximate form

$$\rho(\omega, k) \approx \delta(\omega^2 - k^2 - m_\infty^2) \quad \text{for} \quad k \sim T, \quad (2)$$

where $m_\infty^2 \sim g^2 T^2$ is the leading-order thermal mass (or “asymptotic mass”) of the *hard* excitations. Thus, quite remarkably, the HTL approximation describes correctly the mass-shell behaviour at both soft and hard momenta.

In traditional perturbative calculations of the thermodynamics performed in imaginary time [5], the HTL’s play almost no role: only the Debye mass $m_D \sim gT$ needs to be resummed in the static ($\omega = 0$) electric gluon propagator [4]. Such a simple resummation retains only one moment of the spectral function in eq. (1). Although this is enough to obtain the leading order contribution, $\propto g^3$, of the soft modes to \mathcal{F} , it is clear that this procedure mistreats most of the physical content of the HTL’s.

To overcome this limitation, two approaches have been recently proposed to perform full resummations of the HTL self-energies in the calculation of thermodynamical functions

[8,2]. In Refs. [8], this has been done by merely replacing the free propagators by the HTL-resummed ones in the expression of the free-energy of the ideal gas:

$$\mathcal{F}_0 = \frac{1}{2} \text{Tr} \log D_0^{-1} \longrightarrow \mathcal{F}_{HTL} = \frac{1}{2} \text{Tr} \log(D_0^{-1} + \Pi_{HTL}). \quad (3)$$

In principle, this is just the first step in a systematic procedure which consists in re-summing the HTL's by adding and subtracting them to the tree-level QCD Lagrangian. This would be the extension to QCD of the so-called “screened perturbation theory” [9], a method which, for scalar field theories, has shown an improved convergence indeed, in two- and three-loop calculations. But in its one-loop approximation in eq. (3), this method over-cludes the leading-order interaction term $\propto g^2$ (while correctly reproducing the order- g^3 contribution), and gives rise to new, ultimately temperature-dependent, ultraviolet divergences and associated additional renormalization scheme dependences.

Our approach on the other hand [2] is based on self-consistent approximations using the skeleton representation of the thermodynamic potential which takes care of overcounting problems automatically, without the need for thermal counterterms.

3. The (approximately) self-consistent entropy

Specifically, we consider the 2-loop self-consistent (or “ Φ -derivable” [10]) approximation to the thermodynamic potential \mathcal{F} , and focus on the entropy¹, which in this approximation takes a simple, effectively one-loop, expression [with $N(k_0) = 1/(e^{\beta k_0} - 1)$] :

$$\mathcal{S} = - \int \frac{d^4 k}{(2\pi)^4} \frac{\partial N}{\partial T} \left\{ \text{Im} \ln D^{-1} - \text{Im} \Pi[D] \text{Re} D \right\} \quad (4)$$

but in terms of fully dressed propagators so that $\Pi[D]$ is the one-loop self-energy built out of the propagator D . Thus, any explicit two-loop contribution to the entropy has been absorbed into the spectral properties of quasiparticles. The price to be paid is that Dyson's equation $D^{-1} = D_0^{-1} + \Pi[D]$ becomes an integral equation, which is further complicated by UV problems and, in gauge theories, also by the issue of the gauge symmetry.

In spite of these complications, the expression (4) has some obvious virtues: In addition to its simple quasiparticle interpretation (as the entropy of a non-interacting gas of quasiparticles with effective propagator D), it is manifestly ultraviolet finite (the derivative of the statistical factor acting as an UV cut-off), and provides a non-perturbative approximation to the thermodynamics which is perturbatively correct up to, and including, $\mathcal{O}(g^3)$ (since the neglected 3-loop diagrams start contributing at $\mathcal{O}(g^4)$).

To cope with the problem that Φ -derivable approximations are not gauge invariant in general, we have proposed gauge-independent but only approximately self-consistent dressed propagators as obtained from (HTL) perturbation theory. Using these in eq. (4) gives a gauge-independent and UV-finite approximation for the entropy, which, while being nonperturbative in the coupling, contains the correct leading-order and next-to-leading order effects of the interactions. Both arise from kinematical regimes where the HTL's are justifiable approximations—at hard momenta, they involve the HTL's only in the vicinity of the mass-shell, where the HTL approximation is sound, cf. eq. (2).

¹More generally, on the first derivatives of the thermodynamic potential, like the entropy and — for plasmas with non-zero chemical potential — also the quark density.

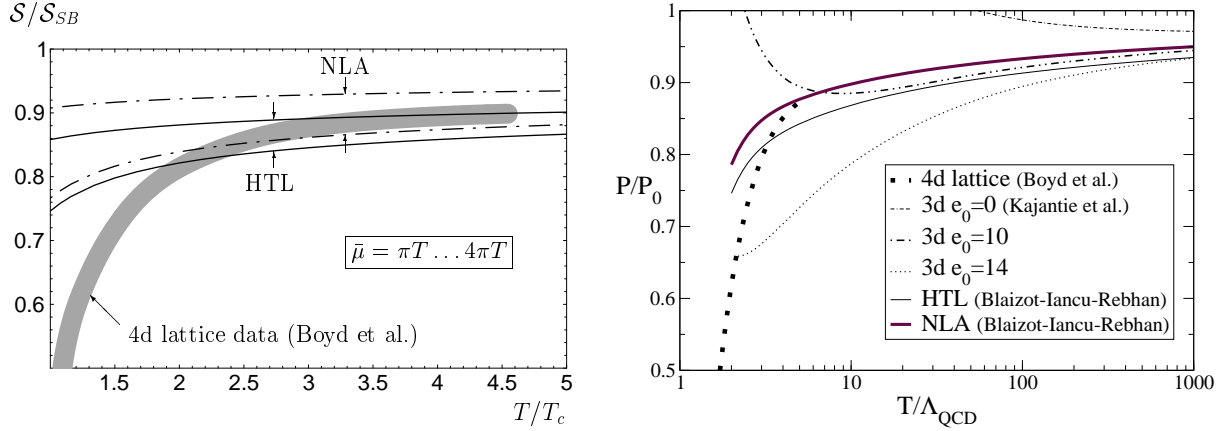


Figure 1. Entropy and pressure for pure-gluon SU(3) Yang-Mills theory: Approximately self-consistent resummation [2] vs. dimensional reduction [3] and 4-d lattice data [1].

In the left half of Fig. 1, the results for the entropy [2] in a pure HTL approximation and in a next-to-leading one (NLA) which includes soft corrections to the hard asymptotic mass are compared with lattice data for pure-gluon SU(3) theory, showing good agreement for $T \gtrsim 2.5T_c$; in the right half, the resulting pressure² is compared in addition to the calculation of the pressure in dimensional reduction of Ref. [3] for temperatures up to $10^3\Lambda_{QCD}$. The fairly good agreement with the best estimates of Ref. [3] ($e_0 = 10$) seems to validate our assumption of comparatively weak residual quasiparticle interactions.

Our method has been applied successfully also to plasmas with non-vanishing quark density (i.e., non-zero chemical potential) [2], for which lattice results are not yet available.

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²with renormalization scheme adjusted so as to match that of Ref. [3]